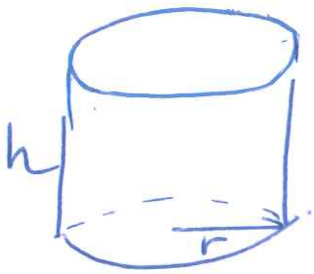
LIBURUKO OPTIMIZAZIOKO BURUKETAK

295/ 9./ DATUAK Zilindroaren azalera $54\ cm^2$

BOLUMENAREN FUNTZIOA maximoa.



$$B(r,h) = \pi r^2 h.$$

$$A = 2\pi r h + 2\pi r^2$$

$$54 = 2\pi r h + 2\pi r^2$$

$$h = \frac{54 - 2\pi r^2}{2\pi r} = \frac{\cancel{2}(27 - \pi r^2)}{\cancel{2}\pi r}$$

$$h = \frac{27 - \pi r^2}{\pi r} \Longrightarrow B(r) = \cancel{\pi} r^{\cancel{2}} \frac{27 - \pi r^2}{\cancel{\pi r}}$$

Beraz optimizatu beharreko bolumenaren funtzioa:

$$\boxed{B(r) = 27r - \pi r^3} \qquad r>0$$

· Bolumena maximoa izateko $B'(r) = 0 \rightarrow B''(r_0) < 0$

$$B'(r) = 27 - 3\pi r^2$$

$$B'(r) = 0 \rightarrow 27 - 3\pi r^2 = 0 \rightarrow r = \sqrt{\frac{27}{3\pi}} = \pm \frac{3}{\sqrt{\pi}}$$

$r = -3/\sqrt{\pi}$ ezin da izan

$r = 3/\sqrt{\pi}$

$$B''(r) = -6\pi r \rightarrow B''\left(\frac{3}{\sqrt{\pi}}\right) = -6\pi \frac{3}{\sqrt{\pi}} < 0 \rightarrow \underline{\underline{\text{MAXIMOA}}}$$

$\boxed{r = 3/\sqrt{\pi}}$ denean Bolumen maximoa hartzen da

$$h = \frac{27 - \pi\cdot(3/\sqrt{\pi})^2}{\pi\ 3/\sqrt{\pi}} = \frac{27 - \frac{9\cancel{\pi}}{\cancel{\pi}}}{\pi\ 3/\sqrt{\pi}} = \frac{6}{\sqrt{\pi}} = \boxed{\frac{6\sqrt{\pi}}{\pi}\ cm = h}$$

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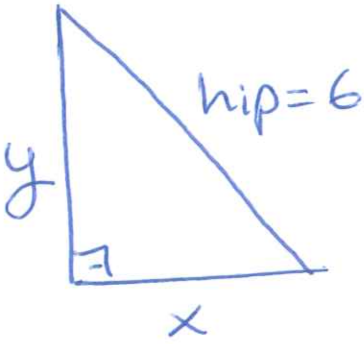
DANA → hipotenusa = 6m.

AZALERAREN FUNTZIOA MAXIMOA



$$A(x,y) = \frac{x \cdot y}{2}$$

DANA → $hip^2 = k_1^2 + k_2^2$

$$6^2 = x^2 + y^2$$

$$y = \sqrt{36 - x^2}$$

- Berat ataleroren funtzioa

$$A(x) = \frac{x \cdot \sqrt{36-x^2}}{2} \qquad x > 0.$$

- Maximoa lortzeko $A'(x)=0$ eta $A''(x_0) < 0$.

$$A'(x) = \frac{1}{2}\left(\sqrt{36-x^2} + x \cdot \frac{-\cancel{2}x}{\cancel{2}\sqrt{36-x^2}}\right) = \frac{1}{\cancel{2}} \frac{\overset{18}{\cancel{36}} - \overset{x^2}{\cancel{2x^2}}}{\sqrt{36-x^2}} \quad \frac{36-x^2-x^2}{\sqrt{36-x^2}} =$$

$$\boxed{A'(x) = \frac{18-x^2}{\sqrt{36-x^2}}}$$

$$A'(x) = 0 \rightarrow \frac{18-x^2}{\sqrt{36-x^2}} = 0 \rightarrow x^2 = 18$$

$$x = \pm 3\sqrt{2}$$

$$\cancel{x = -3\sqrt{2}}$$

	-6	$-3\sqrt{2}$		$3\sqrt{2}$		6
$A'(x)$			$+$		$-$	
$A(x)$			↗	$3\sqrt{2}$ 4,24	↘	

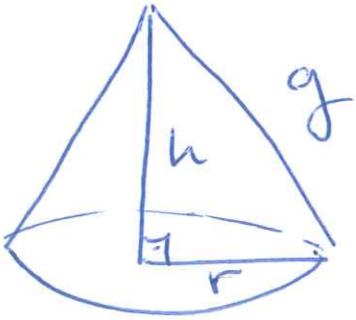
Konprobatzen da $x = 3\sqrt{2}$ funtzioaren maximoa dela.

$$y = \sqrt{36 - (3\sqrt{2})^2} = \sqrt{18} = 3\sqrt{2}.$$

Berat triangeluaren katetoek, biek $3\sqrt{2} = 4,25$ m neurtzen dute

52 | DATUAK : konoaren sortzailea $g = 10\ cm$

BOLUMENAREN funtzioa EDUKIERA MAXIMOA izateko



$$B(r,h) = \frac{\pi r^2 h}{3}$$

DATUA $g = 10\ cm.$

$$g^2 = h^2 + r^2$$

$$100 = h^2 + r^2 \longrightarrow r^2 = 100 - h^2$$

$$B(h) = \frac{\pi \cdot (100 - h^2) \cdot h}{3}$$

$$\boxed{B(h) = \frac{\pi}{3}(100h - h^3)} \qquad h > 0$$

· Maximoa izateko $B'(h) = 0 \longrightarrow B''(h_0) < 0$

$$B'(h) = \frac{\pi}{3}(100 - 3h^2)$$

$$B'(h) = 0 \longrightarrow \frac{\pi}{3}(100 - 3h^2) = 0 \rightarrow h = \sqrt{\frac{100}{3}} = \pm\frac{10}{\sqrt{3}}$$

$h = -10/\sqrt{3}$ ezin da izan.

· konprobatzeko $h = 10/\sqrt{3}$ maximoa dela.

$$B''(h) = \frac{\pi}{3}(-6h)$$

$$B''\left(\frac{10}{\sqrt{3}}\right) = \frac{\pi}{3} \cdot \left(-6\frac{10}{\sqrt{3}}\right) < 0 \longrightarrow$$ Beraz $h = \frac{10}{\sqrt{2}}$ denean <u>MAXIMOA</u> dago.

$$\cdot\ h = \frac{10}{\sqrt{3}} \longrightarrow r = \sqrt{100 - \left(\frac{10}{\sqrt{3}}\right)^2} = \sqrt{\frac{300 - 100}{3}} = \sqrt{\frac{200}{3}} = \frac{10\sqrt{6}}{3} cm$$

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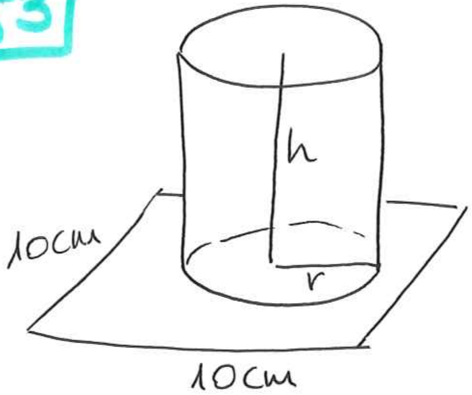


DATUAK: Zilindroaren alboko azalera $50\ cm^2$

BOLUMENAREN FUNTZIOA MAXIMOA izatiko.

$B = \pi r^2 \cdot h.$

$B(r,h) = \pi r^2 h.$

Alboko azalera $A_{alb} = 2\pi r \cdot h \longrightarrow 50 = 2\pi r h \longrightarrow h = \frac{50}{2\pi r}$

$\longrightarrow h = \frac{25}{\pi r}.$

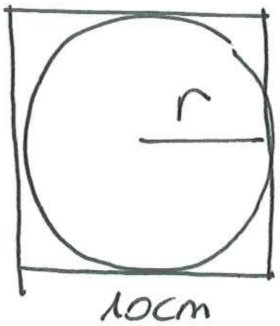
Beraz BOLUMENAREN FUNTZIOA:

$B(r) = \cancel{\pi} r^2 \frac{25}{\cancel{\pi} r} \Longrightarrow \boxed{B(r) = 25r}$

Maximoa kalkulatzeko: $B'(r) = 0 \longrightarrow B''(r_0) < 0.$

$B'(r) = 25 \longrightarrow$ Funtzioa beti gorakorra da, ez dauka maximo erlatiborik.

Beraz maximo absolutua, oinarriko korrotiak baldintzatuko dau.



Beraz $\boxed{r = 5\ cm.}$

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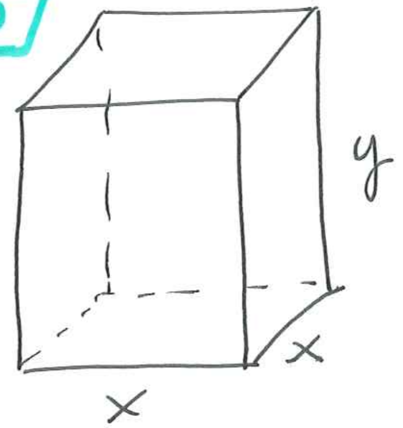

DATUAK: EDUKIERA 80 cm³.

FUNTZIOA OPTIMIZATZEKO: PREZIOAREN FUNTZIOA, MINIMOA izatea

· Oinarriaren materialaren prezioa, tapa eta alboko azala baino %50 garestiagoa dela materiala.

Demagun "p" dela tapa eta alboko azalaren prezioa/m²

· taparen eta alboko azala → p €/cm²

· oinarriaren prezioa → $1{,}5 \cdot p$ €/cm²

· Beraz azalen prezioa:

$$F(x,y) = 1{,}5p \cdot x^2 + p \cdot x^2 + p \cdot 4 \cdot xy$$

$$F(x,y) = 2{,}5p\,x^2 + 4p\,xy$$

· Datua erabiliz: (bolumena = 80 cm³)

$$B = x^2 y \longrightarrow 80 = x^2 y \longrightarrow \boxed{y = \frac{80}{x^2}}$$

· Azaleren prezioaren funtzioa, x-ren menpe.

$$F(x) = 2{,}5p\,x^2 + 4p\cancel{x} \cdot \frac{80}{x^{\cancel{2}}}$$

$$\boxed{F(x) = 2{,}5p\,x^2 + \frac{320p}{x}}$$

· Minimoa lortzeko → $F'(x) = 0$ eta $F''(x) < 0$.

$$F'(x) = 5px - \frac{320p}{x^2}$$

$$5px - \frac{320p}{x^2} = 0$$

$$p\left(5x - \frac{320}{x^2}\right) = 0 \nearrow \quad 5x = \frac{320}{x^2} \rightarrow x^3 = \frac{320}{5} = 64$$

$$\boxed{x = 4} \Rightarrow \boxed{y = 5}$$

minimoa.

F'	$-$	$+$
F	↘ 4	↗

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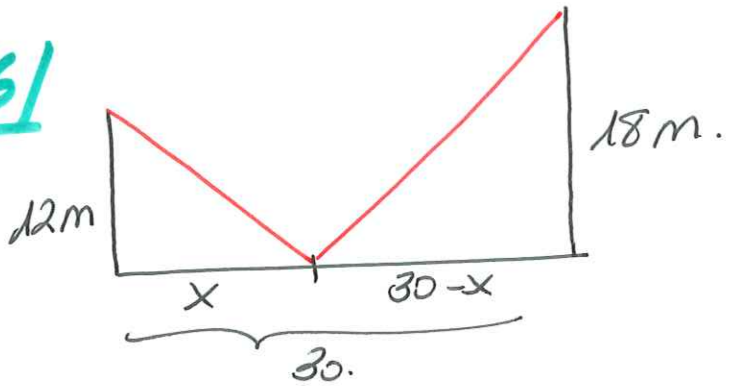
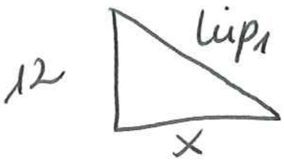
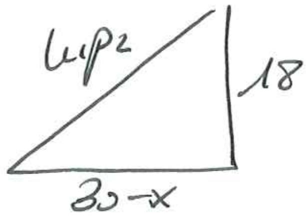


DANAK: Irudian.

FUNTZIOA: kablearen luzera minimoa



$$hip_1=\sqrt{12^2+x^2}$$
$$hip_2=\sqrt{18^2+(30-x)^2}.$$



$$L(x)=\sqrt{144+x^2}+\sqrt{324+900-60x+x^2}$$

$$\boxed{L(x)=\sqrt{144+x^2}+\sqrt{1224-60x+x^2}}$$ Luzeraren funtzioa.

· Minimoa lortzeko $\longrightarrow L'(x)=0 \longrightarrow L''(x)>0.$

$$L'(x)=\frac{\cancel{2}x}{\cancel{2}\sqrt{144+x^2}}+\frac{\cancel{2}x-\cancel{60}\,30}{\cancel{2}\sqrt{1224-60x+x^2}}=$$

$$=\frac{x\cdot\sqrt{x^2-60x+1224}+(x-30)\sqrt{x^2+144}}{\sqrt{(x^2+144)(x^2-60x+1224)}}$$

$$L'(x)=0 \Longrightarrow x\sqrt{x^2-60x+1224}+(x-30)\sqrt{x^2+144}=0$$

$$\left(x\sqrt{x^2-60x+1224}\right)^2=\left((30-x)\sqrt{x^2+144}\right)^2$$

$$x^2(x^2-60x+1224)=(30-x)^2(x^2+144)$$

$$x^4-60x^3+1224x^2=(900-60x+x^2)(x^2+144)$$

$$\cancel{x^4}-\cancel{60x^3}+1224x^2=900x^2+129600-\cancel{60x^3}-8640x+\cancel{x^4}+144x^2$$

$$180x^2+8640x-129600=0$$

$$x^2+48x-720=0.$$

$$x=\frac{-48\pm\sqrt{48^2-4\cdot1\cdot(-720)}}{2}=\begin{cases}12\\ \cancel{-60}\end{cases}$$

		12	
L'	⊖		⊖
L	↘		↗

MINIMOA da 12 metrotako distantzian

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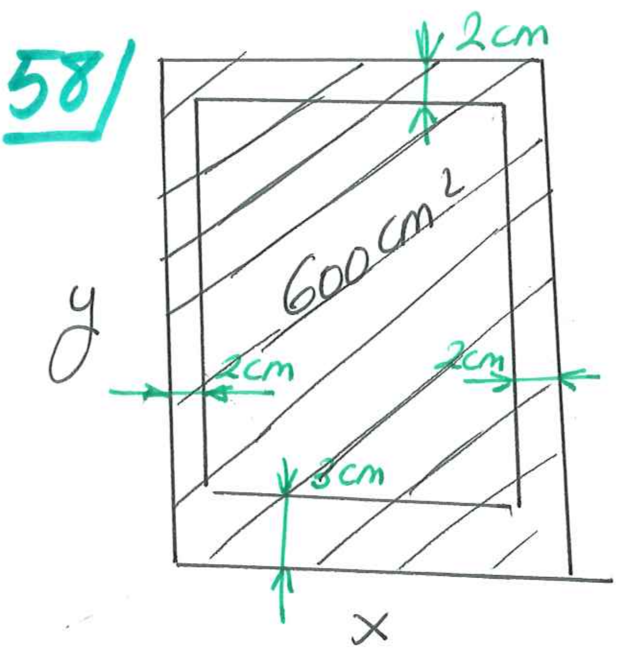

FUNTZIOA: INPRIMATUKO AZALERA MAXIMOA izatiko

DATUA $A = 600 cm^2$

Inprimatuko azalera.

$$A(x,y) = (x-4)\cdot(y-5)$$

$$600 = xy \rightarrow y = \frac{600}{x}.$$

$$A(x) = (x-4)\cdot\left(\frac{600}{x} - 5\right)$$

$$A(x) = 600 - 5x - \frac{2400}{x} + 20$$

$$\boxed{A(x) = 620 - 5x - \frac{2400}{x}}$$

· Maximoa izatiko $A'(x) = 0 \longrightarrow A''(x) < 0.$

$$A'(x) = -5 + 2400/x^2$$

$$0 = -5 + \frac{2400}{x^2} \Longrightarrow \frac{2400}{x^2} = 5 \quad x = \pm\sqrt{\frac{2400}{5}} = \pm 4\sqrt{30}$$

$x = -4\sqrt{30}$ ez dau balio.

· Konprobatzeko $x = 4\sqrt{30}$ maximoa dan.

$$A''(x) = \frac{-2400\cdot 2}{x^3} \longrightarrow A''(4\sqrt{30}) = \frac{-2400\cdot 2}{(4\sqrt{30})^3} < 0 \quad \text{Maximoa da.}$$

· Beraz orrialdearen neurriak izango dira

$$x = 4\sqrt{30} = 21,90 \text{ cm}$$

$$y = \frac{600}{4\sqrt{20}} = 27,39 \text{ cm.}$$